Pressure Change Accompanying Mixing of Two Ideal Gases

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Nomenclature

C_p = specific heat at constant pressure
C_v = specific heat at constant volume
h = enthalpy per unit mass
K = C_p/C_v
m = mass
M = molecular weight
n = number of moles
P = pressure
Q = heat
R = gas constant for a specific gas
$\bar{R}$ = universal gas constant
T = temperature
u = internal energy per unit mass
U = internal energy
V = volume
W = work

Subscripts

1,2 = designates different gases
f = final condition
i = initial condition
m = mixture
T = tank

Introduction

THIS investigation arose because of an interest in the magnitude of the pressure change accompanying the mixing of two gases when one is used to pressurize a container holding the other. If the two gases are separated by a flexible adiabatic membrane, the final pressure differs in general from that resulting if they are allowed to mix. A few simple cases are considered first since they are related to the problem of interest and serve as a basis for comparison. The ideal gas law is assumed along with constant specific heats.

Analysis, Case 1

The problem is to determine the final pressure after an extensible, impermeable, adiabatic barrier between two gases is removed. In general, the gases are different and their initial temperatures differ but their pressures are equal. Figure 1 provides a schematic of the physical situation under consideration.

The basic equations used are as follows:

$$U_1 = m_1u_1 = (P_iV_1/R_1T_1)C_{v1}T_1 \tag{1}$$

$$U_2 = m_2u_2 = (P_iV_2/R_2T_2)C_{v2}T_2 \tag{2}$$

$$U_f = (m_1 + m_2)C_{vm}T_f = P_f(V_1 + V_2)C_{vm}T_f/R_mT_f \tag{3}$$

$$U_f = U_1 + U_2 = P_f(V_1 + V_2)C_{vm}/R_m = (P_iV_1C_{v1}/R_1) + (P_iV_2C_{v2}/R_2) \tag{4}$$

Solving for the pressure after mixing,

$$P_f = [R_mP_i/C_{vm}(V_1 + V_2)][(V_1C_{v1}/R_1) + (V_2C_{v2}/R_2)] \tag{5}$$

Now write C_{vm} and R_m in the forms

$$C_{vm} = (m_1C_{v1} + m_2C_{v2})/(m_1 + m_2) \tag{6}$$

$$R_m = \bar{R}/M_m \tag{7}$$

where

$$M_m = (n_1M_1 + n_2M_2)/(n_1 + n_2) \tag{8}$$

If the ideal gas law is used to replace the n_1 and n_2 of Eq. (8), the gas constant for the mixture can be written from Eq. (7) as

$$R_m = \bar{R}(V_1T_2 + V_2T_1)/(V_1T_2M_1 + V_2T_1M_2) \tag{9}$$

Use of Eqs. (6) and (9) in Eq. (5), along with some algebraic manipulation and use of the ideal gas law, yields

$$\frac{P_f}{P_i} = \left[\left(1 + \frac{C_{v2}M_2V_2}{C_{v1}M_1V_1}\right)\Big/\left(\frac{T_2}{T_1} + \frac{C_{v2}M_2V_2}{C_{v1}M_1V_1}\right)\right] \times \left[\left(\frac{T_2}{T_1} + \frac{V_2}{V_1}\right)\Big/\left(1 + \frac{V_2}{V_1}\right)\right] \tag{10}$$

From this equation it is apparent that when $(T_2/T_1) = 1$, the result is $(P_f/P_i) = 1$. Thus, there is no pressure change due to mixing when the initial temperatures are the same, even if the gases are different. Nor is there any pressure change if the gases are the same and the initial temperatures different. Only if the gases are different and their initial temperatures also different will there be a pressure change associated with the mixing.

Since later cases will require use of the mass ratio m_2/m_1, it is desirable to put Eq. (10) on that basis rather than a volume basis;

$$\frac{P_f}{P_i} = \left[\left(1 + \frac{C_{v2}T_2m_2}{C_{v1}T_1m_1}\right)\Big/\left(1 + \frac{C_{v2}m_2}{C_{v1}m_1}\right)\right] \times \left[\left(1 + \frac{M_1m_2}{M_2m_1}\right)\Big/\left(1 + \frac{T_2M_1m_2}{T_1M_2m_1}\right)\right] \tag{11}$$

The gases of interest are oxygen and helium; thus, the following sample problem is considered. Gas 1: O_2, V_1 = 1000

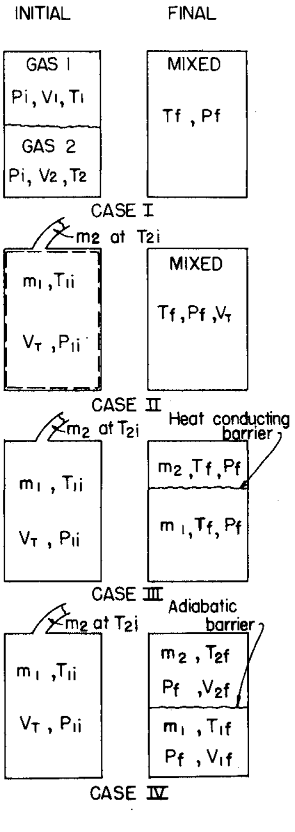

Fig. 1 Schematic of problems considered.

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ft³, $T_1 = 165°\text{R}$, $C_{v1} = 0.155$ Btu/lb °F, $M_1 = 32$. Gas 2: He, $V_2 = 500$ ft³, $T_2 = 500°\text{R}$, $C_{v2} = 0.75$ Btu/lb °F, $M_2 = 4$. Use of this data in Eq. (11) yields results as illustrated in Fig. 2a.

Analysis, Case 2

The schematic is shown in Fig. 1. The problem is to determine the final pressure in the tank after charging with a mass m_2 at a different temperature and then allowing the gases to mix completely.

If potential and kinetic energy terms are neglected, application of the conservation of energy law to the control volume shown by dotted lines in Fig. 1, yields $Q - W = -m_2h_{2i} + (m_2u_{2f} + m_1u_{1f}) - m_1u_{1i}$. However, since the surface of the control volume is assumed adiabatic and no work crosses that surface, $Q = 0$ and $W = 0$. Hence,

$$m_2(h_{2i} - u_{2f}) = m_1(u_{1f} - u_{1i})$$

With complete mixing, $T_{2f} = T_{1f} = T_f$. Since the specific heats have been taken as constants and the gases are assumed ideal,

$$T_f = \frac{m_2C_{p2}T_{2i} + m_1C_{v1}T_{1i}}{m_2C_{v2} + m_1C_{v1}}, \quad P_{1f} + P_{2f} = P_f \tag{12}$$

Substituting expressions for P_{2f} and P_{1f} from the ideal gas law and eliminating T_f by use of Eq. (12), the following relation results:

$$\frac{P_f}{P_{1i}} = \left[\left(1 + \frac{C_{p2}T_{2i}m_2}{C_{v1}T_{1i}m_1}\right)\Big/\left(1 + \frac{C_{v2}m_2}{C_{v1}m_1}\right)\right]\left[1 + \frac{M_1m_2}{M_2m_1}\right] \tag{13}$$

The results for this case are shown in Fig. 2b as plotted from Eq. (13) for the same sample problem as in case I.

The values of (P_f/P_{1i}) from Fig. 2b seem quite large. For example, with a temperature ratio of 0.9 and a mass added equal to that originally in the tank $(m_2/m_1) = 1$, the ratio (P_f/P_{1i}) is 12.8. This might be surprising since the mass in the tank has only doubled. The reason for the large pressure rise is the great differences in specific heats and molecular weights between oxygen and helium. If oxygen were used to pressurize a tank containing oxygen with $(m_2/m_1) = 1$ and $(T_{2i}/T_{1i}) = 0.9$, Eq. (13) yields $(P_f/P_{1i}) = 2.26$, which is about what one might expect.

Analysis, Case 3

The schematic is shown in Fig. 1. Here the problem is to predict the pressure in the tank after charging where the gases are kept separate by an extensible, impermeable, heat conducting membrane. Thus, the gases do not mix but do come to the same temperature and pressure.

After charging with mass m_2, the gas on each side of the barrier is at the same pressure and temperature. Recalling the results of case 1, these properties would not change if the barrier were removed. But if the barrier were removed and the gases allowed to mix, the final condition would be the same as the final condition in case 2. However, since pressure and temperature would not change with membrane removal, the final conditions for case 3 (barrier not removed) must be the same as the final conditions of case 2.

Analysis, Case 4

Here the gas originally in the tank and that used for charging are separated by an extensible, impermeable, adiabatic barrier. Therefore, the gases do not mix or attain the same temperature, but their final pressures are equal. The situation is illustrated in Fig. 1.

Since the barrier is extensible, $P_{1f} = P_{2f} = P_f$. Application of the control volume approach to energy conservation is the same as for case 2. When the specific heats are taken as constants and if the charging gas is assumed to compress gas 1 isentropically, using the ideal gas law and algebraic manipulations, one obtains

$$\left(\frac{P_f}{P_{1i}}\right)^{\frac{1-K}{K}}\left[\left(\frac{C_{v2}}{C_{v1}}\right)\left(\frac{M_2}{M_1}\right)\left(\frac{P_f}{P_{1i}}\right) - \left(\frac{C_{p2}}{C_{v1}}\right)\left(\frac{T_{2i}}{T_{1i}}\right) \times \left(\frac{m_2}{m_1}\right) - 1\right] = \left(\frac{C_{v2}}{C_{v1}}\right)\left(\frac{M_2}{M_1}\right) - 1 \tag{14}$$

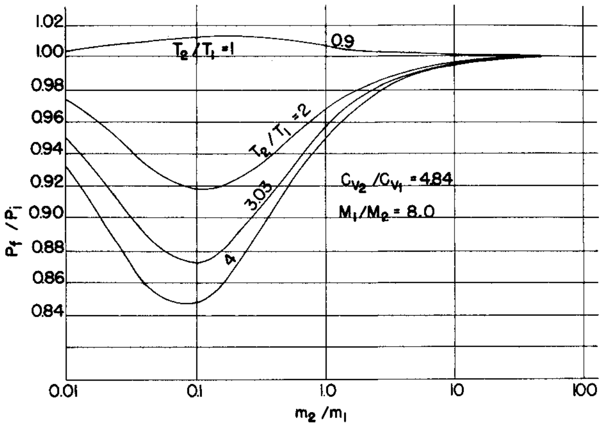

Fig. 2a Results for case 1.

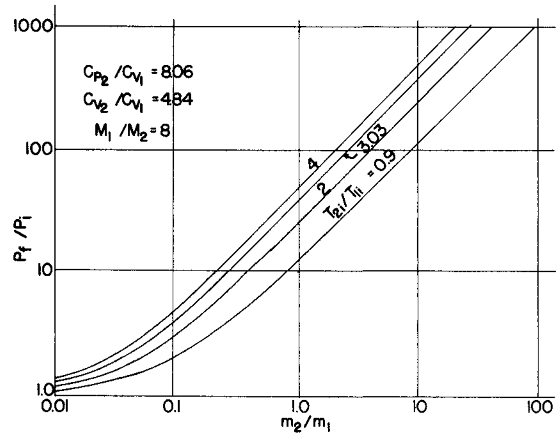

Fig. 2b Results for cases 2 and 3.

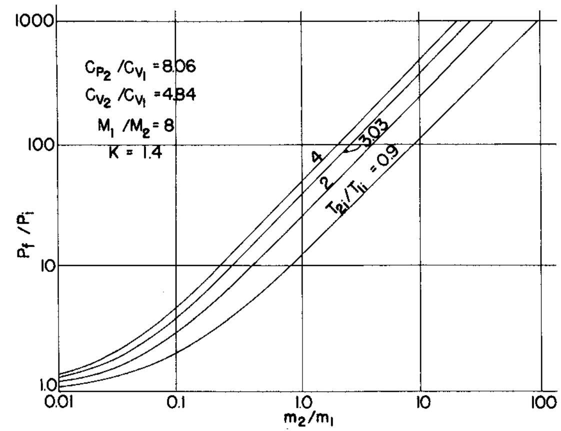

Fig. 2c Results for case 4.

The same sample problem will be used as before. The only new factor is K, and here $K = C_{p1}/C_{v1} = 1.40$.

For given values of the other dimensionless ratios in Eq. (14), (P_f/P_{1i}) can be found from that equation. The results of this procedure are shown in Fig. 2c. As in case 2, (P_f/P_{1i}) varies almost linearly with (m_2/m_1) for large values of that

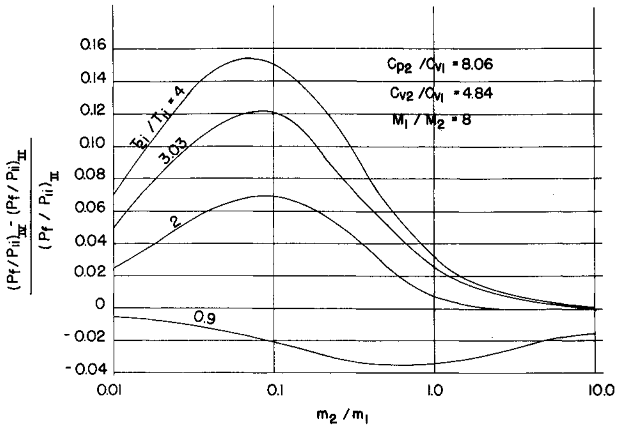

Fig. 3 Comparison of results for cases 2 and 4.

mass ratio. To show this analytically, write Eq. (14) as

$$\left(\frac{C_{v2}}{C_{v1}}\right)\left(\frac{M_2}{M_1}\right)\left(\frac{P_f}{P_{1i}}\right) - \left(\frac{C_{p2}}{C_{v1}}\right)\left(\frac{T_{2i}}{T_{1i}}\right)\left(\frac{m_2}{m_1}\right) - 1 = \left[\left(\frac{C_{v2}}{C_{v1}}\right)\left(\frac{M_2}{M_1}\right) - 1\right]\left(\frac{P_f}{P_{1i}}\right)^{\frac{K-1}{K}} \tag{15}$$

Now for large values of (m_2/m_1), the unity term on the left-hand side and the entire right-hand side of the equation are negligible. Solving for the pressure ratio from the remaining equation yields

$$P_f/P_{1i} = (C_{p2}/C_{v2})(T_{2i}/T_{1i})(M_1/M_2)(m_2/m_1)$$

Not only is this a linear relation, but it is the identical relation found for case 2.

If the barrier in case 4 were removed, the end state would be the same as that of case 2, assuming that the initial conditions were the same in both cases. Thus, the change in pressure in going from the end state of case 4 to the end state of case 2 can be associated with the mixing. In this manner the change in pressure due to charging and that accompanying mixing can be separated and the latter evaluated. A plot of the change in pressure accompanying mixing is given in dimensionless form in Fig. 3. Interest in results such as these provided the initial stimulus for this investigation.

It is of interest to examine the four special situations discussed in the section on case 1. These are a) two gases are the same and $T_{2i}/T_{1i} = 1$; b) two gases are the same and $T_{2i}/T_{1i} \neq 1$; c) two gases are different and $T_{2i}/T_{1i} = 1$; and d) two gases are different and $T_{2i}/T_{1i} \neq 1$.

It was found in case 1 that only (d) yielded a change in pressure upon mixing. Now examine these four situations with regard to the pressure change associated with mixing in the charging arrangement, i.e., that in going from the end state of case 4 to the end state of case 2.

For situation (a), Eq. (13) yields $P_f/P_{1i} = 1 + (C_p/C_v)(m_2/m_1)$, whereas Eq. (15) yields the same result. Thus, the change in pressure accompanying the removal of the barrier thereby going from the final state of case 4 to the final stage of case 2 would be zero.

Now examine situation (b). Equation (13) becomes

$$P_f/P_{1i} = 1 + (C_p/C_v)(T_{2i}/T_{1i})(m_2/m_1)$$

This result is also found by use of Eq. (15). Thus, if the two gases are the same, there will be no pressure change associated with the mixing regardless of the initial temperatures.

In general, there will be a pressure change accompanying the mixing process for situations (c) and (d).

High-*g* Resistant Electronic Fuse for Projectile Payloads

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GUN-LAUNCHED projectiles have placed payloads at high altitudes[1] for sounding, seeding, and performance experiments,[2,3] creating a need for a 60,000 *g* time-delay fuse with delays as long as several minutes. Electronic delay circuits with high-*g* components became available during the last decade, can be temperature compensated, assembled with ease, calibrated, pretested, and manufactured at costs comparable to pyrotechnic devices. This Note describes the development of such fuses.

Fuse Design

Electrical

In 1967 a very sensitive, programable, unijunction transistor (PUT)[4] became commercially available. This transistor, the D13T2, has excellent voltage sensitivity, low leakage, capability for operation down to 2 v d.c., is inexpensive and comes in a solid plastic T0-98 case.

The timing circuit (Fig. 1) was designed around the D13T2, a PNPN device which operates as a triode thyristor. It has a gate-controlled diode section which becomes highly conductive at the appropriate anode-gate voltage. Once the diode section begins to conduct, the regenerative characteristics of the PNPN junction cause the D13T2 to switch on. At the same time a high energy pulse appears at the cathode and can be used to trigger other devices such as a silicon-controlled rectifier (SCR).

The circuit in Fig. 1 can provide time delays up to several minutes by changing the R_1C_1 time constant. The D13T2 turns on when the anode potential is 0.41 v higher than the gate. The gate potential is determined by the battery and the ratio of resistances in the divider network. The anode potential is directly the potential across capacitor C_1. After power is applied, the capacitor C_1 begins to charge and the anode voltage increases until it exceeds the intrinsic standoff potential. Then current begins to flow from anode to gate. The diode section of the D13T2 begins to conduct, regeneration takes place and the PUT is switched on. A fast-rising,

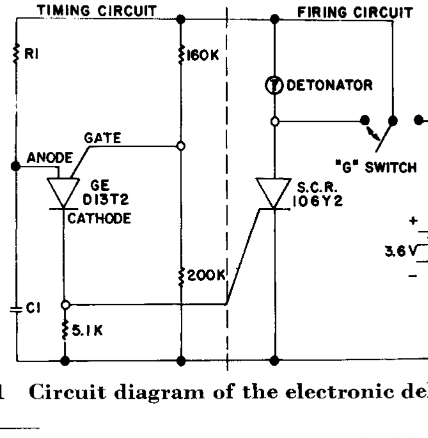

Fig. 1 Circuit diagram of the electronic delay fuse.

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